Thermoelastic loss in microscale oscillators

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A simple model of thermoelastic dissipation is proposed for general, free standing microelectromechanical (MEMS) and nanoelectromechanical (NEMS) oscillators. The theory defines a flexural modal participation factor, the fraction of potential energy stored in flexure, and approximates the internal friction by assuming the energy loss to occur solely via classical thermoelastic dissipation of this component of the motion. The theory is compared to the measured internal friction of a high Q mode of a single-crystal silicon double paddle oscillator. The loss at high temperature (above 150 K) is found to be in good agreement with the theoretical prediction. The importance of this dissipation mechanism as a function of scale is briefly discussed. We find that the relative importance of this mechanism scales with the size of the structure, and that for nanoscale structures it is less important than intrinsic phonon–phonon scattering. © *2002 American Institute of Physics.* [DOI: 10.1063/1.1449534]

Obtaining high Q performance in Si based oscillators in microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS) is desirable for applications that include charge detectors,[1] radio-frequency filters,[2] magnetic resonance force microscopes,[3] and torque magnetometers.[4] Achieving this goal has proven to be difficult however, and to date, the achieved Qs in MEMS/NEMS oscillators have been orders of magnitude smaller than expected from considerations of fundamental loss mechanisms.[5] Explanations of the observed behavior include attachment losses,[6,7] bulk defects,[4] losses associated with the electrical contacts,[8] and surface effects.[9] Unfortunately, untangling the various potential mechanisms is not a straightforward exercise because of the difficulty of controlling and measuring the properties of microscale and nanoscale devices.

The MEMS oscillators are typically free standing, planar samples vibrating predominantly either in flexure or torsion. In the case of a simple beam in flexure, thermoelastic (TE) dissipation is known to be a significant loss mechanism near room temperature.[10] The loss mechanisms in other oscillators involving torsional vibration, or in general possessing a nontrivial mode shape, are however still unknown. We have developed a simple model which predicts the internal friction arising from TE dissipation in these types of oscillators. We have found the internal friction can be quite significant even for nominally torsional vibration modes which one might conclude have no TE loss. Analysis of the experimental results for a macroscopic single-crystal silicon oscillator yields good agreement with the prediction.

TE dissipation arises from thermal currents generated by compression/decompression in elastic media. Typically, the TE dissipation arising from longitudinal waves in solids is very small[11] owing to the large distance between the compressed and rarefied regions. In contrast, the compression associated with flexural vibration can give rise to TE dissipation orders of magnitude larger, because the distance involved in the heat transfer, the sample thickness, is typically orders of magnitude smaller than the wavelength of longitudinal waves, thus resulting in a much higher thermal phonon current and energy dissipation.

Our model of TE dissipation is based on the observation that the resonant modes of elastic structures almost always contain some flexural component. Provided the sample thickness is very small compared to the wavelength, the transverse dimensions are unimportant, and the decay rate associated with the TE dissipation of flexure will be given by

$$\frac{1}{\omega}\frac{dE}{dt}=\phi_f^0 E_f, \tag{1}$$

where ϕ_f^0 is the internal friction associated with pure flexure, E is the total modal energy, and E_f is the modal energy associated with flexure. The assumption underlying Eq. (1) is that the loss rate in an arbitrary vibrational mode is simply given by the loss rate associated with the flexural energy of the mode. The effective modal loss factor due to TE dissipation is thus

$$\phi_f=\phi_f^0\frac{E_f}{E}=p_f\phi_f^0, \tag{2}$$

where $p_f=E_f/E$ is the modal participation factor for flexure.

The loss factor associated with pure flexure, first predicted by Zener,[10] is given by Nowick and Berry[12] as

$$\phi_f^0=\frac{E\alpha^2 T}{c}\frac{\omega\tau}{1+\omega^2\tau^2}, \tag{3}$$

where E is Young's modulus, α the thermal expansion coefficient, c the specific heat per unit volume, T the temperature, and τ the thermal relaxation time. Note that the internal friction is a function of the dimensions of the structure only through the relaxation time τ given by

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$$\tau = \frac{a^2}{\pi^2 D} = \frac{a^2 c}{\pi^2 \kappa}, \tag{4}$$

with a the sample thickness, κ the thermal conductivity, and $D = \kappa/c$ the thermal diffusion constant.

An estimate of the TE loss factor of a general free standing oscillator can be obtained from Eqs. (2) and (3) once the flexural participation factor p_f is known for the particular mode under consideration. This can be determined from the modal displacements as follows.

The potential energy of the vibration is in general given by

$$U = \frac{1}{2} \int dV \sigma_{ij} \epsilon_{ij}, \tag{5}$$

where σ_{ij} and ϵ_{ij} are the stress and strain tensors and repeated indices are to be summed. The flexural part of the potential energy is primarily contained in the $\sigma_{xx}\epsilon_{xx}$ and $\sigma_{yy}\epsilon_{yy}$ terms, where the z axis is chosen to be normal to the plane of the oscillator. The displacements may be Taylor expanded in the thickness direction owing to the smallness of this dimension. The displacement in the x direction, for example, may be expressed as $u_x(x,y,z) \approx u_x(x,y,0) - z\phi(x,y)$ where ϕ is the angle of the normal relative to the z axis. The xx component of the strain is thus

$$\epsilon_{xx} \approx \frac{\partial u_x}{\partial x} - z \frac{\partial \phi}{\partial x} \tag{6}$$

with a similar expression for ϵ_{yy}. The flexural portion of the potential energy consists of the terms containing the derivatives associated with bending,

$$U_f \approx -\frac{1}{2} \int dV z \left(\sigma_{xx} \frac{\partial \phi}{\partial x} + \sigma_{yy} \frac{\partial \phi}{\partial y} \right) \tag{7}$$

and the modal participation factor is thus given by $p_f = U_f/U$.

As the basis for this study, we use the double paddle oscillator (DPO), first introduced by Kleinman.[13] The DPO is distinguished by exhibiting resonances with very high values of Q. These modes are stable and repeatable, have a very small coupling to the environment, and have been previously studied in significant detail.[14,15] Various properties of the DPO, including the dependence of the internal friction on temperature and the mode shapes, are known.[6] The high Q modes of the DPO are thus particularly appropriate for the study of intrinsic sources of energy dissipation in single crystalline Si and serve as a useful test case.

The DPO is shown in the inset of Fig. 1. The oscillator was fabricated from a 300 μm thick, float-zone refined, double-side polished, $\langle 100 \rangle$ oriented, and N-doped silicon wafer with $\sim 2\ \mathrm{k}\Omega\,\mathrm{cm}$ resistivity. We focus on the mode with the highest Q, the second antisymmetric torsion mode[7] in which the head and wings rotate out of phase with one another. The vibration is primarily torsional with the restoring force being provided by the neck. The angular momentum is nearly balanced between the head and the wings, and motion in the foot of the oscillator, the source of loss to the external environment, nearly vanishes.

The measured internal friction of the second antisymmetric mode of the DPO is shown as a function of temperature in Fig. 1 along with the prediction based on Eq. (2). The participation factor for this mode was determined from Eq. (7) using a validated finite element model predictions of the modal displacements[6] and is given approximately by $p_f = 0.0755$. The material properties of Si in this temperature range were taken from the EMIS Datareview *Properties of Silicon*.[16] At low temperatures ($T \sim 10$ K), the internal friction lies approximately at 2×10^{-8}. At 60 K, the loss begins to increase rapidly with temperature and approaches 6×10^{-6} at room temperature. The overall agreement of the predicted TE dissipation with the measured internal friction in temperature dependence as well as in absolute values suggests strongly that TE dissipation due to transverse strains plays a strong role in determining the internal friction for this mode of the DPO at temperatures above 60 K. Indeed, the relaxation frequency at room temperature associated with this mechanism for the DPO is 1,803 Hz. The first seven normal modes[6] fall between 200 and 5500 Hz and are all impacted significantly by TE dissipation.

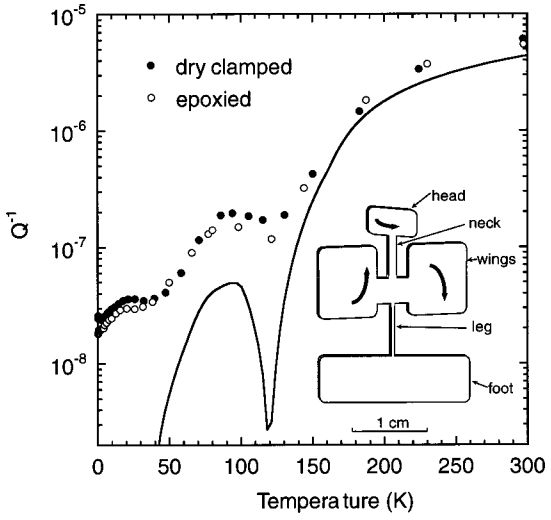

FIG. 1. The measured internal friction of the double paddle oscillator (points) compared to theory (solid line), Eq. (2). The pronounced dip predicted at 120 K is a result of a sign change in α. This is only weakly observed experimentally due to contributions from yet unexplained sources of damping.

The thermodynamic approach underlying the results given above is limited to conditions under which thermal equilibrium is maintained within the sample. We must certainly insist that the phonon mean free path be much smaller than the sample thickness at temperature T, a constraint which in the case of single-crystal Si limits the domain of validity of the method roughly to temperatures $T \geqslant 50$ K. This simple estimate is consistent (albeit perhaps fortuitously) with the failure of the model at low temperatures.

We now consider the scale dependence of this mechanism as it would relate to micro- and nanoscale oscillators based on a planar design. In general, resonance frequencies of linear elastic structures scale inversely with scale factor L,

$$\omega_r \propto L^{-1}, \tag{8}$$

while from Eq. (4) the TE relaxation frequency scales as

$$\tau^{-1} \propto L^{-2}. \tag{9}$$

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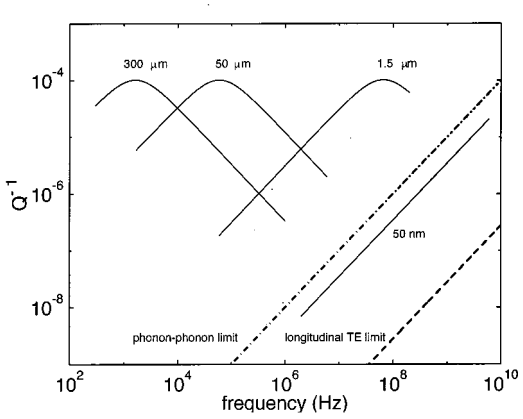

FIG. 2. Internal friction of MEMS/NEMS cantilever beam oscillators. Each curve shows the damping at the fundamental for a beam of fixed thickness and varying length. Also shown on the figure are the internal friction due to phonon–phonon scattering and TE dissipation of longitudinal waves.

Hence, the quantity $\omega_r\tau$, which controls the amount of damping observed at resonance, scales as

$$\omega_r\tau \propto L. \tag{10}$$

Starting with a macroscopic oscillator near its absorption peak ($\omega_r\tau\sim1$), if simple scaling applies, $\omega_r\tau$ and thus TE dissipation become much smaller with a significant reduction in size.

In this vein, it is instructive to consider size effects regarding a simple cantilevered beam as we increase the resonance frequency by shortening the length of the beam from an aspect ratio (length/thickness) of 100:1 down to 5:1. We consider the fundamental bending mode of the beam with a 100% modal participation factor. The internal friction associated with thermoelastic damping at resonance, as predicted by Eqs. (3) and (4), are shown in Fig. 2 for beam thicknesses of 300 μm, 50 μm, 1.5 μm, and 50 nm, respectively. For a given curve, the beam thickness h is fixed while the beam length L varies over the range $(5<L/h<100)$ correspondingly shifting the resonance frequency of the fundamental (the transverse dimension of the beam is irrelevant). The curves indicate that for simple oscillators of this common type, nonscaling reductions in size can lead to both substantial increases and decreases in damping, depending on where one starts with respect to the $\omega_r\tau=1$ point. Our results in this regard are in agreement with those recently obtained by Lifshitz and Roukes,[17] in that TE dissipation may be a relevant mechanism all the way down to the nanometer scale. Examining the above plots, however, it is apparent that some combinations of length and thickness can be found even in these MEMS and NEMS beam oscillators for which TE damping would be quite small. Further, TE dissipation clearly becomes smaller relative to other fundamental dissipation mechanisms as the scale is reduced, becoming smaller than phonon–phonon scattering at the 50 nm scale. However, the thermal equilibrium requirement is almost certainly violated even at room temperature for this scale.

In conclusion, we have proposed a simple model to predict the TE loss of general MEMS/NEMS oscillators. The model successfully predicts the observed internal friction of a high Q macroscopic oscillator similar to free standing MEMS for temperatures above 150 K. For temperatures below this, the observed internal friction is not explained by known loss mechanisms. Further, we have examined the scale dependence of this mechanism and found that the quantity $\omega_r\tau$, which controls the amount of damping observed at resonance, in general scales linearly with resonator size.

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